Choosing a suitable sample size in descriptive sampling [†]

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Abstract

Descriptive sampling (DS) is an alternative to crude Monte Carlo sampling (CMCS) in finding solutions to structural reliability problems. It is known to be an effective sampling method in approximating the distribution of a random variable because it uses the deterministic selection of sample values and their random permutation,. However, because this method is difficult to apply to complex simulations, the sample size is occasionally determined without thorough consideration. Input sample variability may cause the sample size to change between runs, leading to poor simulation results. This paper proposes a numerical method for choosing a suitable sample size for use in DS. Using this method, one can estimate a more accurate probability of failure in a reliability problem while running a minimal number of simulations. The method is then applied to several examples and compared with CMCS and conventional DS to validate its usefulness and efficiency.

Keywords: Crude Monte Carlo sampling; Descriptive sampling; Reliability; Sample size

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1. Introduction

Systems having high performance and confidence levels are in demand. Consequently, estimating a system's probability of failure during the initial design phase is important. The first step in calculating the probability of failure of a structure is to choose random variable $\mathbf{X}=(X_1, X_2, \cdots, X_{n_v})$, where n_v is the number of input variables, and then determine their functional relationship, $Z = g(\mathbf{X})$, which corresponds to the performance criterion. The limit state function, g, of interest can then be defined as $Z = 0$, which is the boundary between the safe and unsafe regions in the design parameter space.

Reliability analysis involves calculating the probability of failure, P_f, which is a multiple integral of the joint probability density function (PDF) on the failure surface of the limit state function. We find that failure occurs when $Z < 0$, and thus the probability of failure is given by the integral

$$P_f = \int \cdots \int_{g(\mathbf{X})<0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x},$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of $\mathbf{X}$. The limit state function plays an important role in a reliability problem. It may be an explicit or implicit function of the input variables and can be written in simple or complicated form. However, in general, finding the PDF of arbitrary random variables is almost impossible. Even if the function can be found, the calculation of the multiple integral is difficult. Thus, studies have been conducted on several methods used for estimating the probability of failure.

One approach to calculating the multiple integral is the use of analytical approximation methods, such as the moment method, which is the most probable point-based method, and the sampling method. The moment method is a method in which one acquires the cumulative distribution function by estimating the moment such as mean, variance and skewness, thus estimating the distribution characteristic or the probability of failure [1-3]. This method, however, requires more experimentations as the number of design variables increases. It also has its limitations on problems with small probabilities.

The most probable point-based method takes the most probable failure point (MPP), which lies on the failure surface and which corresponds to the maximum likelihood of failure occurrence, measuring the probability of failure through this. This method can be grouped into two types: first-order reliability methods (FORM) and second-order reliability methods (SORM). FORM has a limit in accuracy, although is an efficient method to perform reliability analysis. The evaluation of reliability by linear approximation leads to erroneous estimates for limit state functions. The reliability measure is also

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equal regardless of the state of failure surface at about the same MPP. In the case of SORM, if the fitted point is inappropriate, the reliability estimate may also be inaccurate. Furthermore, the SORM approach can obtain the wrong curvature caused by the numerical noise, and the computation time increases when the number of random variables increases [4-8].

Sampling methods, such as crude Monte Carlo sampling (CMCS), importance sampling (IS), and descriptive sampling (DS), are used in the approximation of integration technique. The basic concept of CMCS is simple and can be used to calculate the probability of failure without the transformation of the limit state function. However, this method requires huge samples to obtain an accurate estimator [9-13]. For variance reduction of the estimate of integration, IS is used. It concentrates the distribution of sampling points in the region of most importance. However, the drawback of IS is that the accuracy of the estimate depends on the choice of the sampling density function, and the region of most importance must be known in advance [12, 13].

An alternative approach to CMCS is DS. It is based on a deterministic selection of the input sample values and their random permutations. The usefulness of DS was confirmed in several comparisons conducted by Saliby [14-16]. The estimate obtained by DS is unbiased and has a lower variance than that obtained by the classical method of CMCS [17, 18]. In Ziha [19], DS was applied to IS for the reduction of variance in the probability of failure. It has also been used in experimental design [20, 21] and risk analysis simulation [22, 23].

Applying DS to complex experiments is difficult, and thus the sample size is sometimes determined haphazardly by the user. Sample size may also vary between runs due to input sample variability, leading to poor simulation results. To overcome this problem, a numerical method for choosing a suitable sample size is proposed. By using the proposed method, a more accurate probability of failure can be estimated using a minimal number of simulations.

We present several examples to show the performance of this method using R(ver. 2.6.1), a language and environment for statistical computing [24]. Through these examples, this method is shown to be both useful and efficient.

2. Concept of descriptive sampling

DS was proposed in order to avoid set variability in simulation studies. When using standard simple random sampling or the CMCS approach, two kinds of variation occur in a randomly generated sample: one is related to the set of values and the other to their sequences. The set of values, which is defined by sample values, is taken together but without considering the its particular sequence or the particular sequence in which the values occur. Both features are closely related to the two basic probability concepts: the set of values is expected to display a pattern of relative frequencies in good agreement with the sampled distribution, and the sequence is expected to

display a lack of order of such values or a pattern of randomness. When using simple random sampling, both set and sequence are allowed to vary. Once they vary, simulation estimates will also vary. Therefore, both features can be ascertained as the two sources of variability of simulation estimates [15, 16, 25].

DS is designed to reduce or remove set variability. It is based on the deterministic selection of input sample values and their random permutations. Symbolically, it follows that

$$DS = deterministic\ set \times random\ sequence,$$

while

$$CMCS = random\ set \times random\ sequence.$$

Unlike CMCS, the use of DS requires knowledge of the total sample size, N, which is related to a full simulation run [26]. Once the sample size is known, at least approximately, the set of values can be defined for each input random variable X_j, $j=1,\cdots,n_v$, using the inverse transform method, and thus

$$x_j^i = F_{X_j}^{-1}[(i-0.5)/n_s]\,, \quad i=1,\cdots,n_s\,, \tag{1}$$

where $F_{X_j}^{-1}(R)$, $R \in (0,1)$, is the inverse transform for the particular input distribution, and n_s is the input sample size of each input variable [27].

To complete the DS generation process, each of the sets of input values is used in a random sequence in each simulation run. Unlike in simple random sampling, the set of values is the same for all replicated runs, n_r, in a simulation. This random shuffling process is easily accomplished by sampling the descriptive set of values without replacement [15].

Consider an n_v-dimensional design problem. For each random variable X_j, a one-dimensional descriptive set of input sample size n_s can be generated according to Eq. (1). The total theoretical number of samples in an n_v-dimensional space is

$$n_t = n_s^{n_v}\,.$$

If N-samples are requested, where $n_s \le N \le n_t$, the number of runs is

$$n_r = N/n_s\,. \tag{2}$$

The anticipated input sample size, n_s, lies within 10% of the actual sample size used in a conventional simulation [27]. In reliability problems, the descriptive set has to be large enough to account for the tail of the distribution in the design point, X^*, in

$$F_t = \min_{1 \le j \le n_v} [1 - F_{X_j}(X^*)] \approx [1-(1-P_f)^{1/n_v}]/2\,. \tag{3}$$

The input sample size can be found in Eqs. (1) and (3),

$$n_s = (i-0.5)/F_t\,. \tag{4}$$

The minimal sample size is found by assuming that at least
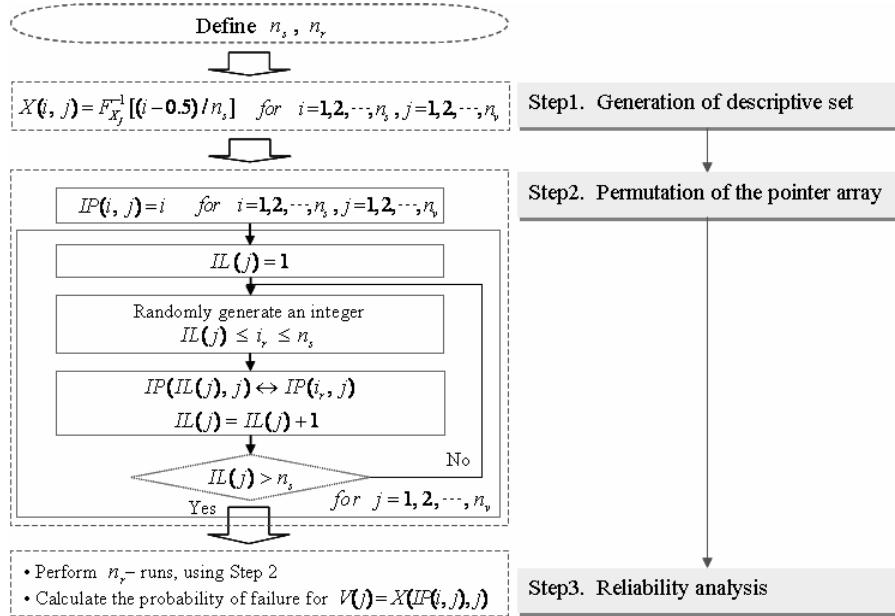
Fig. 1. Flow chart of DS for reliability analysis.

one element of the descriptive set is in the failure region, $i = 1$, in Eq. (4). This is usually not the most efficient sample size, and an adjustment is needed for each particular problem. Hence, the total number of evaluations is defined as

$$N = n_s n_r . \tag{5}$$

Therefore the input sample size and the number of runs have to be calculated and determined to obtain the total sample size.

An explanation of the DS algorithm follows. A set value, $X(n_s, n_v)$, is generated. A pointer array, $IP(n_s, n_v)$, for the permutation of indices instead of set values is then determined. $IL(n_v)$ is the integer vector, pointing to the first available $IP(n_s, n_v)$.

Fig. 1 shows the flow chart of this algorithm. In Step 1, a descriptive set value, $X(n_s, n_v)$, is generated by Eq. (1). In Step 2, a pointer array is randomly permutated for each variable in Step 1. The probability of failure for the current run is then calculated in Step 3.

For example, if samples in $N = 10$, $P_f = 0.79$, and $n_v = 7$ are used, the input sample size and the number of runs are calculated to be $n_s = 5$ and $n_r = 2$ from Eqs. (2-4). Table 1 presents randomly permutated pointer values and descriptive sample set values for a seven-dimensional problem, in which $N = 10$, $n_v = 7$, $n_s = 5$, and $n_r = 2$. For $n_r = 1$ and $n_r = 2$, the five pointer values are generated by random permutation without replacement. The set values in Table 1 show that the descriptive sample set is generated according to Eq. (1) from the standard normal distribution, $N(0, 1)$.

3. Numerical method for the determination of sample size

As shown in Chapter 2, the total number of evaluations in a reliability problem is determined by the number of input vari-

ables, the input sample size, and the number of replicated runs. Of these parameters, the number of input variables is most easily determined because it is related to the problem definition. Therefore, to adapt DS to real problems, only the input sample size and the number of replicated runs must be determined. However, in many cases, these have been chosen without thorough consideration by the user, who does not know how to determine them. Moreover, because the accuracy of the results is not guaranteed, applying this method to complex simulations is difficult. A suitable input sample size and the number of iterations are required. In a failure probability computation, a "suitable" sample size is the minimum sample size for an acceptably accurate result.

A numerical method for the determination of a suitable sample size in DS, is proposed. The method has three stages: 1) the probability of failure is approximated; 2) the input sample size is determined using the approximated probability of failure; and 3) the number of iterations is determined to obtain the total sample size.

3.1 Approximation of the probability of failure

In Eqs. (2) and (3), we observed that the probability of failure must be calculated to obtain the input sample size. However, calculating the probability of failure is not possible without an input sample size because all simulations require input points. Hence, an approximation of the probability of failure must be made. At this stage, the approximation is found using several crude Monte Carlo samples.

The numerical method for the approximation of the probability of failure is illustrated in Fig. 2. First, the lower limit of the number of failures, n_{f_0}, and the lower limit of the probability of failure, P_{f_0}, are initiated. These are the criteria used in choosing if the approximated probability of failure is rea-

Table 1. Example of a shuffled set of pointer array for $N = 10$, $n_v = 7$, $n_s = 5$, and $n_r = 2$, and the set values from the standard normal distribution $N(0,1)$.

				Point array								Set values			
n_r	n_s $\backslash$ n_v	1	2	3	4	5	6	7	1	2	3	4	5	6	7
	1	5	3	5	3	1	1	4	1.28	0.00	1.28	0.00	-1.28	-1.28	0.52
	2	2	1	1	5	3	4	5	-0.52	-1.28	-1.28	1.28	0.00	0.52	1.28
1	3	4	4	2	4	2	5	1	0.52	0.52	-0.52	0.52	-0.52	1.28	-1.28
	4	1	2	3	2	5	2	2	-1.28	-0.52	0.00	-0.52	1.28	-0.52	-0.52
	5	3	5	4	1	4	3	3	-0.13	0.00	0.52	-1.28	0.52	0.00	0.00
	1	2	5	1	1	2	3	3	-0.52	1.28	-1.28	-1.28	-0.52	0.00	0.00
	2	4	3	5	3	4	5	2	0.52	0.00	1.28	0.00	0.52	1.28	-0.52
2	3	5	2	4	5	3	1	1	1.28	-0.52	0.52	1.28	0.00	-1.28	-1.28
	4	3	1	3	4	1	2	5	0.00	-1.28	0.00	0.52	-1.28	-0.52	1.28
	5	1	4	2	2	5	4	4	-1.28	0.52	-0.52	-0.52	1.28	0.52	0.52
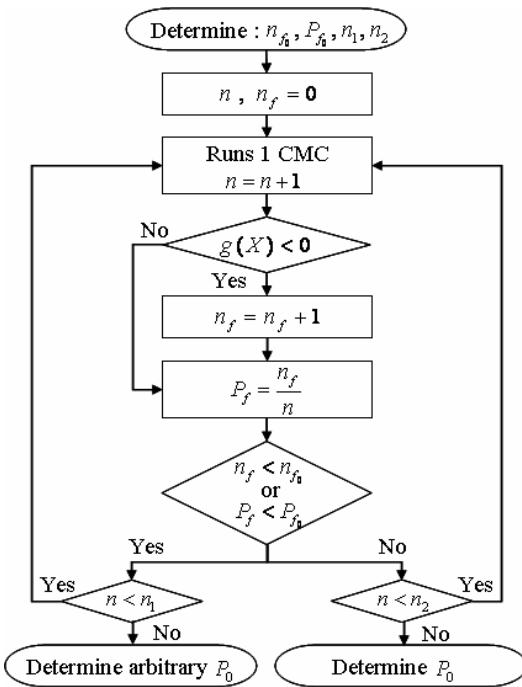
Fig. 2. Flow chart of the approximation of the probability of failure.

Fig. 3. Flow chart of the determination of input sample size.

sonable.

Setting n_{f_0} to be equal to 1, and P_{f_0} to be equal to 0.0001 is proposed. If the general value of P_f is known, P_{f_0} should be set equal to that value. The smallest input sample size, n_1, and the reasonable sample size, n_2, are also initiated to confirm the approximation of the probability of failure. That is, n_1 is used to choose the sampling stopping point that occurs when P_f is not reasonable, and n_2 is used to determine whether P_f has been approximated. Here, that n_1 be set to 100 and n_2 be set to 10 is proposed no matter how complex the simulation.

Subsequently, a CMCS is run. Here, if a CMCS shows $g(X) < 0$, the number of failures, n_f, is increased by one each. That is, n_f is the counting number of failures. A simulation can then be run along with the computation of P_f. This step is repeated until the terminating condition is satisfied.
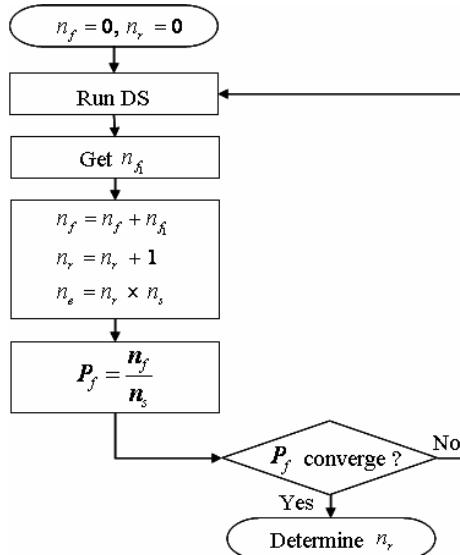
That is, when n_f is less than n_{f_0}, or when P_f is less than P_{f_0}, and if n_s is less than n_1, then go to [Runs 1 CMC] in Fig. 2. Otherwise, the approximated probability of failure is arbitrarily determined to be P_0. When n_f is larger than n_{f_0}, P_f is larger than P_{f_0}, and n_s is less than n_2, then go to [Runs 1 CMC] in Fig. 2. Otherwise, substitute P_f for P_0. In this way, the probability of failure, P_0, can be approximated.

3.2 Determination of input sample size

At this stage, an input sample size can be computed using the approximated probability of failure, P_0. In Eq. (3), F_t can be determined by inserting the value for P_0 computed in Section 3.1; the input sample size is then computed as in Eq. (4). According to Ziha [19], the minimal sample size is obtained by assuming $i = 1$ in Eq. (4), but this assumption is not valid because an element of the descriptive set cannot be in the failure region. One must confirm this assumption before using DS.

In Fig. 3, a numerical method for choosing the input sample
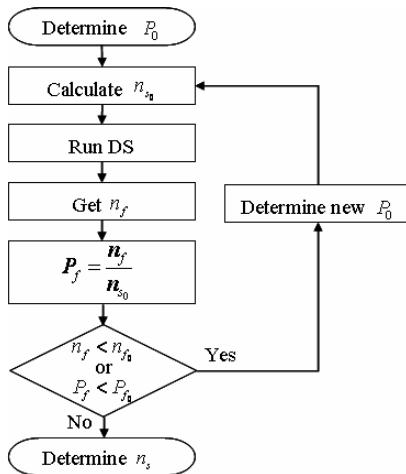
Fig. 4. Flow chart of the determination of the number of runs.

size is suggested. First, when $i=1$ in Eq. (4), the initial input sample size, n_{s_0}, is calculated using P_0 from Fig. 2. DS is then experimented once using n_{s_0} to obtain n_f and P_f. If n_f or P_f is very small, a new probability of failure, P_0, must be determined to find a suitable input sample size, n_s. The values of n_{f_0} and P_{f_0} must be set to a small value. That n_{f_0} be set to 1 and P_{f_0} be equal to 0.0001 is proposed, as stated previously. That P_0 be set to a value 0.5 times smaller than the previous value of P_0 is also proposed. Otherwise, there is a need to substitute n_s for n_{s_0} and to terminate the iterations. In this way, a total sample size with a suitable input sample size n_s is obtained.

3.3 Determination of the number of runs

After the input sample size is determined as shown in Section 3.2, the number of runs, n_r, needed to minimize the total sample size while maintaining precision is given by Eq. (5).

Fig. 4 shows a numerical method for determining the total sample size. First, n_f and n_r are set to 0. Next, DS is experimented once with the input sample size, and the number of failures, n_{f_1}, is computed. Then, n_f, n_r and N can be updated. The number of runs, n_r, is found when the values of P_f converge, and the iterations are stopped. To determine when P_f converges, iterations continue until a simulation gives the same result as the previous simulation. At this point, the iterative process is terminated, and P_f is known.

A numerical method to choose the total sample size in DS is proposed. Using this numerical method, one can estimate the probability of failure more accurately with a minimal number of simulations.

4. Simulation

In this section, our numerical method for choosing the total sample size in DS with five mathematical examples using functions of both low and high dimensions is validated. Using these examples, the usefulness and efficiency of the proposed

method can be presented.

In each example, the precision of the probability of failure and the total sample size found using the proposed method are acceptable. For comparisons, DS was run once with the proposed numerical method to determine the input sample size and the number of runs. CMCS was then run using the same total sample size used in DS. The results show that DS compares favorably with CMCS but does not guarantee that the proposed method is superior to the conventional DS method, in which the parameters are determined haphazardly. Hence, a comparison of the proposed method with the conventional method was conducted by running several simulations using the same total sample size in all of them.

The procedure for validating the proposed method was as follows. First, the method was executed once in DS, and the total sample was determined. The total sample size was then altered slightly to compare the proposed method with the conventional method, because DS must be executed with many combinations of input sample sizes and numbers of runs, but with the same total sample size. An altered total sample size can be decomposed as products of two integers, where at least one integer is greater than or equal to 100. In this way, the CMCS method, which is the conventional method, and the proposed method of DS were compared using the same total sample size. These comparisons were done using the results of 100 independent runs and are summarized by the precision, variance, and coefficient of variation of the computed probability of failure.

4.1 Low dimensional examples

These examples are two-dimensional functions formed with convex, concave, and non-convex and non-concave functions. These functions are frequently used for calculating reliability [3]. In these examples, $n_{f_0}=1$, $P_{f_0}=0.0001$, $n_1=200$, and $n_2=20$ are set.

4.1.1 Convex performance function

A convex function is given as

$$g(\mathbf{X}) = -\exp(X_1 - 7) - X_2 + 9,$$

where X_1 and X_2 represent the independent $N(6.0, 0.8^2)$. The exact probability of failure, P_{exact}, is obtained using 10,000,000 crude Monte Carlo runs,

$$P_{exact} = 9.20 \times 10^{-3}.$$

Based on Section 3, the total sample size is determined to be $N=1000$. The input sample size and the number of replicated runs are changed to $n_s = 200$ and $n_r = 5$, respectively, for comparison with the conventional method.

Table 2 shows that the probability of failure found using DS is the same as the exact probability of failure.

The proposed method provides a more accurate probability of failure, a lower variance, and a lower coefficient of varia-

Table 2. Comparison of results for the convex function.

Method	N^a ($n_s \times n_r$)	$P_f \times 10^{-3}$	var$\times 10^{-6}$	c.o.v
CMC	1000	9.24	8.992	0.325
DS	1000×1	9.21	7.016	0.288
	500×2	9.21	4.854	0.239
	200×5 [b]	9.20	4.000	0.217
	100×10	7.86	3.778	0.247

[a] Total sample size $N = 1000$
[b] Result from the proposed method

tion than CMCS. In comparing three results found using the conventional method with different input sample sizes and numbers of runs, $n_s = 1000$ and $n_r = 1$, $n_s = 500$ and $n_r = 2$, and $n_s = 100$ and $n_r = 10$, the proposed method gives a more accurate estimate of the probability of failure.

4.1.2 Concave performance function

A concave function is given as

$$g(\mathbf{X}) = X_1^3 + X_2^3 - 18 ,$$

where $X_1 \sim N(10, 5^2)$ and $X_2 \sim N(9.9, 5^2)$. The exact probability of failure is $P_{exact} = 5.70 \times 10^{-3}$, obtained using 10,000,000 crude Monte Carlo runs.

The input sample size and the number of runs are determined to be $n_s = 350$ and $n_r = 3$, respectively, using the proposed method. Table 3 shows that the probability of failure found using the proposed method converges to the exact probability of failure. The method provides a more accurate estimate than that given by CMCS. This method also gives a lower coefficient of variation compared with any of the results found using DS.

4.1.3 Non-convex and Non-concave function

An example of a non-convex and non-concave function is given by

$$g(\mathbf{X}) = 3 - (X_1 + 0.25)^2 + (X_1 + 0.25)^3 + (X_1 + 0.25)^4 - X_2 ,$$

where X_1 and X_2 are the independent standard normal random variables. The exact probability of failure is obtained using 10,000,000 crude Monte Carlo runs, $P_{exact} = 3.63 \times 10^{-3}$.

In this example, the input sample size and the number of runs are determined to be $n_s = 450$ and $n_r = 13$, respectively.

Table 4 shows that the total number of simulations in DS is greater than that for the convex and concave functions, but the results are similar. The probability of failure found with DS using the proposed method is a good approximation to the exact probability of failure and is more accurate than that found using CMCS. Although the probability of failure in DS using input sample size $n_s = 5850$, and the number of replicated runs $n_r = 1$ is more accurate than that found using the proposed method, the method gives a lower coefficient of variation compared with the other results found using DS.

Table 3. Comparison of results for the concave function.

Method	N^a ($n_s \times n_r$)	$P_f \times 10^{-3}$	var$\times 10^{-6}$	c.o.v
CMC	1050	5.85	5.699	0.408
DS	1050×1	5.70	5.460	0.410
	350×3 [b]	5.70	4.562	0.375
	210×5	5.48	4.350	0.381
	105×10	5.44	4.275	0.238

[a] Total sample size $N = 1050$
[b] Result from the proposed method

Table 4. Comparison of results for the non-convex and non-concave function.

Method	N^a ($n_s \times n_r$)	$P_f \times 10^{-3}$	var$\times 10^{-6}$	c.o.v
CMC	5850	3.52	0.592	0.691
DS	5850×1	3.63	0.482	0.191
	1170×5	3.66	0.324	0.156
	450×13 [b]	3.62	0.290	0.149
	225×26	2.94	0.268	0.176

[a] Total sample size $N = 5850$
[b] Result from the proposed method

4.2 High-dimensional examples

These examples are multi- and complicated functions. One is a function containing both artificial and high-frequency noise; the other is the union of three high-dimensional functions. In these examples, n_{f_0}, P_{f_0}, n_1, and n_2 are set differently than in the previous examples. Simulation with DS using the proposed method is run, followed by that using CMCS and conventional methods.

4.2.1 Function with artificial and High-frequency noise

The following limit state function [28] is selected:

$$\begin{aligned} g(\mathbf{X}) = & X_1 + 2X_2 + 2X_3 + X_4 - 5X_5 - 5X_6 \\ & + 0.0001(\sin(100X_1) + \sin(100X_2) + \sin(100X_3) \\ & + \sin(100X_4) + \sin(100X_5) + \sin(100X_6)) \end{aligned}$$

This function contains artificial and high-frequency noise from six independent random variables with the following log normal distribution:

$$X_1, X_2, X_3, X_4 \sim Lognormal(120, 12^2) ,$$
$$X_5 \sim Lognormal(50, 15^2) , \quad X_6 \sim Lognormal(40, 12^2) .$$

The exact probability of failure calculated with crude Monte Carlo sampling is $P_{exact} = 0.0121$ using the NESSUS Program Version 6.0 [29] with 10,000,000 runs.

By letting $n_{f_0} = 1$, $P_{f_0} = 0.001$, $n_1 = 100$, and $n_2 = 20$, as shown in Figs. 2-4, the input sample size, $n_s = 350$, and the number of replicated runs, $n_r = 45$ are determined. As shown in Table 5, DS using the proposed method gives a bet-

Table 5. Comparison of results for a function with high-frequency and artificial noise.

Method	N^a ($n_s \times n_r$)	$P_f \times 10^{-3}$	var$\times 10^{-6}$	c.o.v
CMC	15750	0.0123	0.898	0.077
DS	15750×1	0.0121	0.530	0.060
	1575×10	0.0122	0.520	0.059
	$350 \times 45^{\,b}$	0.0121	0.495	0.058
	175×90	0.0114	0.550	0.065

[a] Total sample size $N = 15750$
[b] Result from the proposed method

Table 6. Comparison of results for a function which is the union of three high-dimensional functions.

Method	N^a ($n_s \times n_r$)	$P_f \times 10^{-3}$	var$\times 10^{-6}$	c.o.v
CMC	30000	4.94	0.297	0.110
DS	30000×1	4.85	0.081	0.059
	3000×10	4.87	0.073	0.056
	$1000 \times 30^{\,b}$	4.85	0.060	0.051
	300×100	4.68	0.059	0.052

[a] Total sample size $N = 30000$
[b] Result from the proposed method

ter result than CMCS and is more efficient than the conventional method.

4.2.2 Function which is the union of three high-dimensional functions

The plane frame structure from Madsen et al. [30] is considered in this example. The principle of virtual work gives three limit state functions:

F_1sway : $g_1(\mathbf{X}) = X_1 + X_2 + X_4 + X_5 - 5X_6$,
F_2frame : $g_2(\mathbf{X}) = X_1 + 2X_3 + 2X_4 + X_5 - 5X_6 - 5X_7$,
F_3beam : $g_3(\mathbf{X}) = X_2 + 2X_3 + X_4 - 5X_7$.

Plastic moment capacities and loads are lognormally distributed independent variables:

X_1, X_2, X_3, X_4 and $X_5 \sim Lognormal(134.9, 13.49^2)$,

$X_6 \sim Lognormal(50, 15^2)$, $X_7 \sim Lognormal(40, 12^2)$.

The exact probability of failure [30] is

$$P_{exact} = P\left(\bigcup_{i=1}^{3} F_i \right) = 4.85 \times 10^{-3} .$$

By setting $n_{f_0} = 1$, $P_{f_0} = 0.0001$, $n_1 = 300$, and $n_2 = 50$, as shown in Figs. 2-4, the input sample size, $n_s = 1000$, and the number of runs, $n_r = 30$ are determined. Table 6 shows that the probability of failure found using DS is more accurate than that found using CMCS, and the variance in the DS re-

sults is lower. The proposed method also finds a lower variance, a lower coefficient of variation, and a more efficient probability of failure than the other DS results.

For the examples in this chapter, $10 \sim 100$ as the range of n_r is used. The range of n_r is not restricted, but the accuracy of the estimator decreases as n_r increases. Hence, using too large a number of runs, n_r , for the simulations is avoided.

5. Conclusions

In this paper, a numerical method for choosing a suitable sample size is proposed. The usefulness of this method is demonstrated through comparisons with crude Monte Carlo sampling and descriptive sampling in reliability examples. The main results are as follows:

(1) A three-stage numerical method is developed for calculating an accurate probability of failure with a minimal number of simulation runs.
   (a) The approximate probability of failure is computed and iteratively updated with one crude Monte Carlo sampling until the terminating condition is satisfied.
   (b) The input sample size is determined using the computed approximate probability of failure.
   (c) The number of replicated runs required to compute the precise probability of failure is determined.
   In this manner, the suitable total sample size is obtained.
(2) The simulation results show that descriptive sampling using the proposed method is more efficient than the crude Monte Carlo sampling and give a more accurate probability of failure across input sample sizes and a number of runs in given problems.

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